Demand Reduction in Multi-unit Auctions with Varying Numbers of Bidders: Theory and Evidence from a Field Experiment

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Abstract

Recent auction theory and experimental results document strategic demand reduction by bidders uniform-price auctions. The present paper extends this area of research to consider the effects of varying the number of bidders. Our theoretical model predicts that demand reduction should decrease with an increase in the number of bidders. Considerable demand reduction remains even in the asymptotic limit, although truthful bidding yields profits very close to those of equilibrium play. We experimentally confirm several of our predictions by examining bidding behavior of subjects in an actual marketplace, auctioning dozens of sportscards using both uniform-price and Vickrey auction formats.

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1. Introduction

Uniform-price sealed-bid auctions have been used in several important applications, from Treasury bonds to transferable pollution permits to initial public offerings of stock shares. Yet, recent theoretical work[2] on multi-unit auctions has established a potential problem with this auction format: in a uniform-price auction, bidders have an incentive to bid lower than their true values, an effect usually referred to as "demand reduction." This theoretical effect is cause for concern because it may lead to inefficient allocations and considerable revenue losses. Recent experimental evidence demonstrates that significant demand reduction does indeed occur with actual bidders, and that the effect is large enough to influence equilibrium allocations.

In a uniform-price sealed-bid auction, each bidder submits bids on one or more identical units of the same good. If there are m units of the good for sale, then the m highest bids each win, and the $(m+1)$st bid becomes the price paid for each unit won. Vickrey (1961, 1962) examined this auction for the case where each bidder demands no more than one unit of the good. In that case, bidders have a dominant strategy of bidding their values, and the auction is allocatively efficient. The analysis becomes much more complicated when an individual bidder may win more than one unit. As before, each bidder finds it optimal to bid her true value on the first unit that she may win, but this is no longer the case for bids on additional units.[3] In particular, an individual's bid on any unit after the first may prove to be the marginal bid that sets the price she must pay on all units won. This gives the auction a "pay-your-bid" flavor, so bidders have an incentive to bid less than their values on all units except the first—this

[2] Vickrey (1961) pointed out that the uniform-price auction would not be demand-revealing if bidders had multi-unit demand, but only recently have there been characterizations of the equilibria in these auctions.
[3] We restrict our attention to perfect equilibria; Levin (2003) provides an equilbrium in the case of two bidders in which their first bids may exceed their values, but it is not a perfect equilibrium.

phenomenon has been termed "demand reduction."[4] Because bidders may no longer truthfully reveal demand, the uniform-price auction may fail to allocate the goods efficiently.

An alternative auction mechanism that overcomes this nuance is the (generalized) Vickrey auction. The pricing rule for this auction ensures that each winning bidder pays an amount equal to the sum of the losing bids that would have won had she not bid.[5] The dominant strategy is for individuals to bid their true values on all units; hence, there is no demand reduction. In theory, this multi-unit Vickrey auction results in an efficient allocation of goods. The Vickrey auction, however, does have potential drawbacks. In addition to the fact that its rules are relatively difficult to understand, it also entails the possibility of an "unfair" outcome where a low-demand bidder pays more than her high-demand rival, despite the fact that her bids were lower than his.[6] This may occur even to the point where the low-demand bidder pays more for a single unit than her rival does for multiple units.

Given these potential drawbacks of the Vickrey auction relative to the uniform-price auction, a designer of auction mechanisms might wish to know whether demand reduction in uniform-price auctions is a nontrivial problem in practice. Available empirical evidence suggests that bids in uniform-price auctions do exhibit significant demand reduction. For example, Kagel and Levin (2001) have examined the case in which one bidder may win more than one unit. They designed a laboratory experiment where a single human bidder competed for two units of a good against robot bidders with unit demand, and found significant demand reduction. They also provided the first theoretical work on the comparative statics of demand reduction, examining the special case of a single multi-unit bidder competing against *m* single-unit

[4] See Ausubel and Cramton (2002) and Engelbrecht-Wiggans and Kahn (1998) for more formal arguments.
[5] Note that this is a special case of a Groves-Clarke mechanism (Groves 1973, Clarke 1971).
[6] A solution to this problem is found in Ausubel (1997), who proposes an ascending-bid auction that is strategically equivalent to the sealed-bid Vickrey auction for the case of private values, but the rules are much simpler to explain to bidders. His auction does not address the potential problem of perceived "unfairness" of the pricing rule, however.

bidders with uniformly distributed private valuations. They derived a unique Nash equilibrium prediction that as the number of bidders increases, the multi-unit bidder should engage in less demand reduction. Their laboratory experiments show an asymmetric confirmation of this prediction: subjects moving from 3 to 5 rivals do not learn to decrease their demand reduction, but subjects moving from 5 to 3 rivals do exhibit additional demand reduction.

Evidence of demand reduction also exists in cases with two multi-unit bidders. List and Lucking-Reiley (2000) found evidence of demand reduction in a field experiment, auctioning nearly $10,000 worth of sportscards in two-unit, two-person sealed-bid auctions. They found that individuals tend to reduce bids on second units by as much as 73 percent in a uniform-price auction relative to a theoretically demand-revealing Vickrey auction. Wolfram (1998) provides evidence of the same strategic effect in uniform-price electricity supply auctions in England and Wales, with two major firms competing against each other.[7]

While these studies provide important insights, whether their results readily transfer to auctions with more than two bidders who can win more than one unit is largely unknown. Many important practical auctions—for example, those for pollution permits, FCC spectrum licenses, and Treasury bills—have more than two bidders who may win multiple units. Does demand reduction vanish as the number of bidders increases, and if so, how quickly? Katzman (2004) provides theoretical arguments that some demand reduction remains even in the limit of infinitely many bidders. His asymptotic arguments, however, do not inform us how much demand reduction, or how quickly the limit is approached. Furthermore, empirical evidence remains scant.

[7] Technically there are more than two competitors in the Welsh electricity auctions, but Wolfram explains that the market consists of two major players plus a competitive fringe of smaller players.

In this study, we provide insights into these issues by focusing on the uniform price auction in which the price equals the highest losing bid.[8] For simplicity and tractability, we restrict attention to the case where exactly two units are for sale; we consider variation in the number of bidders but not the number of units. We begin by deriving new theoretical predictions for the uniform-price auction, examining demand reduction's dependence on the number of bidders. Specifically, we propose a model with privately known, decreasing marginal values, independent across bidders.

For our model, it follows easily from the results of Engelbrecht-Wiggans and Kahn (1998) that as the number of bidders changes, equilibrium bids change in a particular fashion. Specifically, the equilibrium second-unit bid for any value v must be either zero or some amount, $c(v)$, where $0 < c(v) < v$ with probability one and $c(v)$ is independent of the number of bidders. Accordingly, a change in the number of bidders only influences the set S of values for which the equilibrium bid is zero (versus nonzero). Furthermore, since $c(v)$ is independent of the number of bidders and is less than v with probability one, the second-unit bid cannot converge to truthful bidding as the number of bidders goes to infinity. In other words, demand reduction does not go to zero as the number of bidders goes to infinity.[9] Unfortunately, the theory does not convey whether the amount of demand reduction (or, equivalently, the probability measure of the second-unit values for which the second-unit bid is zero) increases or decreases with the number of bidders. Neither does it quantify the amount of demand reduction.

To answer these questions, we numerically solve approximately 500 examples to illustrate how the set S of zero-bid values changes as the number of bidders increases. In each of our examples, the set S of zero-bid-values decreases (weakly) as the number of bidders

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[8] In practice, the lowest accepted bid has often been used to determine the price instead of the highest rejected bid. Similar arguments are true for that alternative uniform-price auction.

[9] However, as Swinkels (1996) has shown, demand reduction is zero in the limit of infinitely many bidders.

increases.[10] As a result, the equilibrium second-unit bid strategy increases (weakly) with the number of bidders. Therefore, at least in all of our examples, demand reduction (weakly) decreases with the number of bidders. Furthermore, in each of our examples either the second-unit bid is zero with a probability of at least 95%, or the second-unit bid is less than 1/30 of the second-unit value on average. In short, equilibrium theory predicts dramatic demand reduction in every example that we consider.

Our examples suggest that at equilibrium, in each case the difference in the number of zero bids (and therefore the absolute difference between the two types of auctions) shrinks as the number of bidders increases, but stays bounded away from zero. And, because of the dominant strategy of truthtelling in the Vickrey auction, changes in the number of bidders should not affect bidding behavior in this auction institution. Hence, in our empirical exploration we consider differences between the Vickrey and uniform-price auctions in terms of: (1) mean second-unit bidding behavior, (2) proportion of second-unit bids equaling zero, and (3) proportion of auctions where both goods are allocated to the same bidder.

Our experimental investigation explores these issues associated with demand reduction in auctions with three or five multi-unit bidders bidding for two units. The methodology follows that of the two-bidder auctions of List and Lucking-Reiley (2000), with participants in a competitive marketplace participating in sealed-bid auctions for collectible sportscards.[11]

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[10] This monotonicity of the set of zero bids does not follow obviously from the results of Engelbrecht-Wiggans and Kahn (1998), and indeed we have been unable to establish such a result more generally than by numerically solving examples.

[11] Our field experiment differs from laboratory experiments in that it involves subjects bidding for real objects in a pre-existing market. It therefore represents a joint test of both the underlying modeling assumptions (weakly decreasing individual demand for multiple units, common knowledge, and symmetry of the distribution of bidder values) and the behavioral assumptions (Nash equilibrium bidding). By contrast, laboratory experiments typically impose the modeling assumptions, and concentrate on clean tests of the behavioral assumptions. In order to gain the ability to perform field tests of both types of assumptions, the experimenter must give up some of the controls of the laboratory. In particular, we give up being able to control (or even observe) bidders' valuations for the good, which makes it impossible to compare observed bids to bidders' true values directly. Therefore, we focus on testing theoretical predictions that compare behavior across two different auction formats: uniform-price and Vickrey.

Although the field data do not perfectly match every theoretical prediction, our experimental results are generally consonant with the theory's various conjectures. In particular, we find evidence of demand reduction, but the level of demand reduction is much smaller and more difficult to detect than it was in the case of $n = 2$ bidders. Differences in mean bids between Vickrey and uniform-price auctions are typically not statistically significant, but proportions of zero bids are significantly different across auction formats. Allocations of goods also differ between auction formats in the predicted direction, but usually not in a statistically significant manner.

We organize the remainder of the paper as follows. In the next section, we develop a theory that links the amount of demand reduction with the number of bidders. In Section 3, we describe our field experiment designed to test these hypotheses in auctions involving three and five bidders. Section 4 discusses the plausibility of the model's assumptions in the sportscard market. Section 5 describes our experimental results, and Section 6 concludes.

2. Demand Reduction in Theory

2.1 The Model

Two identical units will be auctioned (with a reserve price of zero) to n expected-profit-maximizing bidders with privately known, decreasing marginal values independent across bidders.[12] Specifically, bidder i has a value v_{i1} for winning one unit and a marginal value of v_{i2} ($v_{i2} < v_{i1}$ with probability one) for winning a second unit. Bidder i knows the values v_{i1} and v_{i2}. The value vectors (v_{11},v_{12}), (v_{21},v_{22}) ... (v_{n1},v_{n2}) are the outcomes of n independent draws from some

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[12] We only consider a relatively small number of bidders and hold the number of units fixed at two. This is not to suggest that demand reduction is merely an academic curiosity with little practical relevance because we usually have a large number of bidders. Indeed, if demand reduction decreases with the number of bidders but increases with the square of the number of units, then demand reduction may be a very serious problem. The interested reader should see Swinkels (2001), who argues that the effect of demand reduction on the auction outcome goes to zero at equilibrium in the limit of infinitely many bidders and infinitely many units.

commonly known, bivariate distribution. Let G_1 and G_2 denote the marginal distributions, and g_1 and g_2 the corresponding densities, where g_1 has support $[0, V]$.

In the auction, each bidder i submits two bids b_{i1} and b_{i2}, and the two highest bids win. As previously mentioned, bidders in the Vickrey auction have a dominant strategy of bidding equal to their values on each of the units. In the uniform-price auction, Engelbrecht-Wiggans and Kahn (1998) argue that any equilibrium in undominated strategies has bidders bidding equal to their true values on the first unit. This leaves only the question of how bidders should bid on the second unit in uniform-price auctions.

In Appendix 1, we examine existing theory for insights into the relationship between second-unit equilibrium bids and the number n of bidders. Herein, we observe that existing theory implies that equilibria depend on n in a very specific fashion. In particular, the first order condition is independent of n. Therefore, as n changes, the equilibrium bid on the second unit may change from being zero to being positive (or vice versa), but if the bid is positive then it is independent of n. The question then becomes "What does the set of second unit values such that the second bid is zero look like, and how does it vary with n?" One possibility is that the equilibrium bidding strategy $b(v;n)$ for a second unit of value v in an auction with n bidders is zero if $v \leq v^*(n)$, and is equal to $c(v)$ otherwise, where $c(v)$ is some function independent of n (namely, the solution to the first order condition), and $v^*(n)$ is a critical value that may change with the number of bidders. We derive sufficient conditions for such "simple monotonic" equilibria. We also argue that the second bid is positive with probability one (and is therefore independent of n) if the marginal density of the first unit value is infinity when the value is zero, and such that the second bid is zero with probability one for all n (and therefore also independent of n) if the marginal density of the first unit value is monotonically increasing.

Our theory facilitates the computation of equilibria for specific examples. In some cases, the theory assures that the second-unit bid will be zero with probability one, or will be positive with probability one and determined solely by the first order condition. More generally, our theory provides a systematic procedure for finding candidate equilibria and for verifying whether or not they are in fact equilibria. Then, once we have computed the equilibria, we can examine how they vary with n.

2.2 Examples

Let v_{i1} and v_{i2} be the larger and smaller of two independent draws from a distribution, $H(x)$; then $G_1(x) = H^2(x)$ and $G_2(x) = 1\text{-}(1\text{-}H(x))^2$. In this way, $H(x)$ is a simple way to give rise to the specified marginal distributions. But, there are many other (infinitely many) joint distributions of v_{i1} and v_{i2} that give rise to the same marginal distributions and, therefore, also the same equilibria.[13] We considered two families of distributions for $H(x)$. These two families include a wide variety of different specific distributions. For the case of values from a distribution with bounded support, we consider the Beta(α_1, α_2) distribution; for the case of values whose distribution is unbounded above, we consider the Gamma(α, β = 1) distribution.[14] The Beta distribution can be used to approximate a wide variety of density functions with bounded support, including uni-modal (symmetric and asymmetric), hook shaped, and, of course, the uniform. Specifically, we consider the Beta distribution with α_1 and α_2 equal to (all combinations of) ⅓, 1, 2, 3, 5, 7, and

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[13] For example, each bidders' values could be the larger and smaller of two independent draws from a uniform distribution. In this case, the joint distribution of v_{i1} and v_{i2} is uniform over the triangular region below the main diagonal of the unit square. Consider any rectangle (with sides parallel to the axes) within this triangular region and redistribute the probability mass from within the rectangle uniformly along either diagonal of the rectangle. The result is a distinctly non-uniform distribution over the triangle, but with the same marginal distributions as before.

[14] The Beta distribution with parameters α_1 and α_2 has density function: $f(x;\alpha_1,\alpha_2) = \{x^{(\alpha_1 -1)}(1\text{-}x)^{(\alpha_2 -1)}\}/\beta(\alpha_1,\alpha_2)$ for $0 < x < 1$ and $f = 0$ otherwise, where $\beta(\alpha_1,\alpha_2)$ is the Beta function defined by $\beta(\alpha_1,\alpha_2) = \Gamma(\alpha_1)\,\Gamma(\alpha_2)/\,\Gamma(\alpha_1+\alpha_2)$; and where $\Gamma(z)$ is the usual gamma function, defined as $(z\text{-}1)!$ if z is a positive integer and in general as the integral:

$$\int_0^\infty t^{(z-1)} e^{-t}\, dt$$

The Gamma distribution with parameters α and β has density function $f(x;\alpha,\beta) = \{\beta^{(-\alpha)}\, x^{(\alpha\, -1)}\, e^{(-x/\beta)}\}/\Gamma(\alpha)$ for $x > 0$ and zero otherwise, where $\Gamma(z)$ is the gamma function defined above.

10. The Gamma distribution can be used to approximate a wide variety of different uni-modal densities that have tails to the right, including the exponential. Specifically, we consider the Gamma distribution with α equal to 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 2, 3, 4, 5, 7, and 10; the β parameter simply rescales the distribution, so, without loss of generality, we always set it equal to 1.0. For each of the distributions, we consider the cases of n = 2, 3, 4, 5, 6, 7, 8 and 9 bidders. In total, this provides approximately 500 different examples.

In each of our examples the first order condition has a unique solution that is weakly increasing in v.[15] Furthermore, the necessary conditions in the Theorem (in Appendix 1) are always satisfied. As a result, for each of our examples, there is exactly one equilibrium that satisfies the sufficient conditions of our Theorem.[16] In all cases, the second-unit bid is positive if and only if the value is above some cutoff. Therefore, for any distribution, the equilibrium for all n will be characterized by the cutoff as a function of n together with the solution to the first order condition. And, in all cases, the cutoff is a weakly decreasing function of n.

Tables 1a and 1b present detailed results for the case when $H(x)$ is a Beta distribution with $\alpha_1 = 1$.[17] When $\alpha_2 \leq 1$, $g_1(x)$ is a strictly increasing function of x, and therefore satisfies the condition of Corollary 2 (in the Appendix) for a single-unit bid equilibrium. Indeed, the tables show that the second-unit bid is zero with probability one.[18] Table 1a reveals how the cutoff is a weakly decreasing function of n for other values of α_2. As the number of bidders increases, the

[15] There do exist examples without uniqueness or monotonicity. Specifically, Engelbrecht-Wiggans (1999) provides an example in which $C'(v) = [0, v]$ for all v. This example has a continuum of equilibria, and this set of equilibria is independent of n. Engelbrecht-Wiggans and Kahn (1998) provide an example in which the first order condition has a unique solution, but it is not monotonic; they also show how a continuum of equilibria may be constructed in such cases.

[16] Uniqueness is a sticky issue. In particular, Engelbrecht-Wiggans and Kahn (1998) do not claim that their procedure finds all of the equilibria—there could be others. But, we are unaware of any perfect equilibria that cannot be obtained by their procedure.

[17] The case where $\alpha_1 = 1$ and $\alpha_2 = n = 2$ is example 3 in Engelbrecht-Wiggans and Kahn (1998).

[18] A Beta distribution with both parameters equal to one gives the uniform distribution. In the case where each bidder's values are the larger and smaller of two independent draws from a uniform distribution, Chakraborty and Engelbrecht-Wiggans (2001) argue that the single-unit bid equilibrium is the only equilibrium in measurable strategies.

equilibrium bid for bidder i may change from $(v_{i1}, 0)$ to $(v_{i1}, c(v_{i2}))$. But that is the only possible change. Figure 1 graphically displays two possible bid functions for the distribution with $\alpha_2 = 3$. For $n = 2$ bidders, one finds that the bid function begins at $b(v) = 0$ for all $v < v^*$, then jumps to $c(v^*) > 0$ at v^*, and remains on the function $c(v)$ for all $v > v^*$. For $n = 5$ bidders, one can readily see that the appropriate v^* moves leftward, so that fewer zero bids result when there are more bidders, but the $n = 5$ bid function otherwise looks identical to the $n = 2$ bid function.

The peculiar form of the bidding strategy's dependence on n also provides insights into the limiting case of infinitely many bidders. As n increases, the amount of demand reduction might strictly decrease, with $v^*(n)$ approaching zero as n approaches infinity. However, $c(v) < v$ with probability one, so the limit of $b(v;n)$ is also less than v with probability one. In other words, some demand reduction persists even in the limit of infinitely many bidders.[19]

Table 1a suggests that the cutoff does not converge to zero for our examples with $\alpha_1 = 1$. More dramatically, note that in Table 1b the probabilities of zero second-unit bids seem to converge to something much closer to one rather than zero. Indeed, the cutoff need not converge to zero as n goes to infinity. For the cutoff to equal zero, the first order condition must have a (at least one) solution for all possible second-unit values. And, in each of our examples, the first order condition fails to have a solution below some critical value. This critical value below which the first order condition no longer has a solution is a lower bound on the limit of the cutoff as n goes to infinity. The probability of the second-unit value being below this critical value is a lower bound on the probability of the second-unit bid being zero. We calculated these bounds for each of the distributions that we considered. They do indeed bound the results obtained for the various

[19] Katzman (2004) establishes the asymptotic persistence of demand reduction without first characterizing the equilibria.

different n. And, in the interest of saving space, we report only these bounds for the remaining examples.

Tables 2a and 2b summarize the results for all the cases of the Beta distribution by presenting the above-mentioned lower bounds in the cutoff and the probability of a zero second-unit bid. In particular, when $\alpha_1 \geq 1$ and $\alpha_2 \leq 1$, Corollary 2 assures that the second-unit bid is zero with probability one, and the tables illustrate this point. For all other cases, when $\alpha_1 \geq 1$, there is a positive probability that the second-unit bids will be positive. But, as Table 2b shows, the probability of the second-unit bid being zero never drops below 0.99 in any of the examples with $\alpha_1 \geq 1$.

If $\alpha_1 < ½$, then Corollary 1 assures that the second-unit bid is always positive. This is illustrated by the examples, reported as "$\alpha_1 < ½$" in the table, for which α_1 was set to ⅓. Note that although there is zero probability that the second-unit bid will be zero, there is still significant demand reduction. Indeed, as Table 3 shows, on average, the second-unit bid is always less than 1/40 of the second-unit value.

Finally, Tables 4a and 4b summarize results for cases in which $H(x)$ is a Gamma (α, β = 1) distribution. Results are qualitatively similar to those for the Beta distributions. In particular, Corollary 1 assures that the second-unit bid will be positive with probability one when α is less than one half. In these cases, Table 4b shows that the second-unit bid stays below 1/30 of the second-unit value. Otherwise, the second-unit bid is zero with positive probability; Table 4a shows that the probability of the second-unit bid being zero is at least 0.95 whenever the probability is positive, and is at least 0.99 whenever $\alpha \geq 1$.

2.3 Deviations from Equilibrium

Real bidders may deviate from equilibrium bidding for a variety of reasons. For example, what if some intuitively appealing, simple bidding strategy performs nearly as well as the equilibrium bidding strategy? Then a bidder may be satisfied with using the slightly sub-optimal simple strategy.[20] In the case of uniform price auctions, a typical conjecture is that bidding truthfully on each unit is a dominant strategy. This is a particularly simple strategy; for example, there is no need to estimate important features, such as the number of other bidders, and adjust one's bid accordingly. We now investigate how much a bidder's expected revenue suffers from bidding, for example, truthfully on both units rather than the best response to others' strategies.

We focus on a specific example. Specifically, assume that each bidder's two values have the same marginal distributions as they would if the values were the larger and smaller of two independent draws from the standard uniform distribution (ie, Beta(1, 1) distribution). Recall that in this case, Chakraborty and Engelbrecht-Wiggans (2001) show that the only Nash equilibrium in measurable strategies has each bidder bidding truthfully on the first unit and zero on the second unit. And, indeed, this is a bidder's best response regardless of how other bidders bid on their second units.

How much does a bidder suffer from bidding truthfully on both units rather than following the Nash equilibrium? Consider two cases: 1) All other bidders bid truthfully on both units, and 2) All other bidders follow the equilibrium strategy. Let $\Pi_{A,B}$ denote the expected profit to a bidder who follows strategy A when all others follow strategy B, where A and B are each either T(truthful on both units) or E(equilibrium). Coupling theorem 2.4 of Engelbrecht-

[20] Indeed, if one adds a cost of learning or using more complex strategies to the model, then the simpler strategy may in fact become the equilibrium. For example, Chakraborty and Engelbrecht-Wiggans (2001) have recently suggested a model in which there is a (small) cost to bidding optimally rather than truthfully. At equilibrium, bidders bid truthfully if it is not worth their while to incur the extra cost of bidding optimally. As the number of bidders goes to infinity, the equilibrium probability of truthful bidding goes to one.

Wiggans and Kahn (1998) with some straightforward though lengthy calculations provides the following results:

$\Pi_{T,T} = 3/((2n+1)n)$,

$\Pi_{E,T} = (6n-1)/((2n+1)n(2n-1))$,

$\Pi_{T,E} = 4(3n-4)/((2n+1)(2n-1)(2n-3))$,

$\Pi_{E,E} = 2(6n-7)/((2n+1)(2n-1)(2n-3))$.

Thus, if everyone else bids truthfully on both units, a bidder loses $\Pi_{E,T} - \Pi_{T,T} = 2/((2n+1)n(2n-1))$ from bidding truthfully rather then optimally (i.e., zero) on the second unit. If everyone else bids according to the equilibrium predictions, then a bidder loses $\Pi_{E,E} - \Pi_{T,E} = 2/((2n+1)(2n-1)(2n-3))$ from bidding truthfully rather than optimally (ie, zero) on the second unit.

As might be expected, the expected profits approach zero with the square of the number of bidders. But, note that the expected loss from truthful, rather than optimal, bidding goes to zero with the cube of the number of bidders. In particular, the loss as a proportion of total profits goes to zero as the number of bidders goes to infinity. For example, if everyone else bids truthfully, then the proportional loss is $(\Pi_{E,T} - \Pi_{T,T})/ \Pi_{E,T} = 2/(6n-1)$ = 0.1818, 0.1176, 0.08696 and 0.06897 respectively when n = 2, 3, 4 and 5. Similarly, if everyone else bids zero on the second unit, then the proportional loss is $(\Pi_{E,E} - \Pi_{T,E})/ \Pi_{E,E} = 1/(6(n-1)-1)$ = 0.2, 0.0909, 0.0588 and 0.04343 respectively when n = 2, 3, 4 and 5.[21]

2.4 Summary

In each of our examples, the conditions for simple monotonic equilibria turn out to be satisfied. In some cases, the cutoff is zero for all n, the bid function is independent of n, and therefore the amount of demand reduction is also independent of n. In other cases, the cutoff is at

[21] Chakraborty and Engelbrecht-Wiggans (2001) have subsequently argued more generally that the loss from bidding truthfully rather than optimally on the second unit shrinks rapidly as the number of bidders increases.

the top of the support for all n, the second-unit bids are zero with probability one, but the equilibrium bids are again independent of n. In these two cases, the amount of demand reduction is independent of n. In all cases, if the cutoff varies with n, it decreases as n increases and, therefore, the amount of demand reduction decreases as n increases. In short, in all our examples, the amount of demand reduction turns out to be a non-increasing function of n.

All of the examples exhibit extreme demand reduction. Specifically, whenever the second-unit bid has positive probability of being zero, that probability is at least 0.95, and is at least 0.99 except for the examples using a Gamma distribution with shape parameter $\alpha < 1$. In some cases, the second-unit bid is positive with probability one, but in each of these cases, on average, the second-unit bid is less than 1/30 of the second-unit value. In sum, our theory predicts extreme demand reduction. This may, however, be a very weak prediction. In particular, the effect of sub-optimal bidding on the second unit may be small enough so that it does not concern a practical bidder. Real bidders may have a pre-disposition to bid positively on both units—for example, to bid truthfully on both units—and we suggest that they suffer relatively little from doing so, especially as the number of competitors increases. Therefore, we do not necessarily expect—and indeed do not find—that actual bids exhibit demand reduction to the extent predicted by the equilibria to our examples.

3. Experimental Design and Procedures

We design a field experiment to explore demand reduction in multi-unit auctions with more than two bidders, and examine how the amount of demand reduction varies with the number of bidders. We begin with data on the two-bidder auctions of List and Lucking-Reiley (2000),[22] and extend the data by conducting auctions with three and five bidders using a similar

[22] We choose to reproduce data from only the two high-value ($70) cards from the early treatments: a Cal Ripken, Jr. 1982 *Topps* card and a Barry Sanders 1989 *Score* card. The two low-value ($3) cards produced similar effects,

experimental design. In the new experimental treatments, we auction three different types of Cal Ripken, Jr. rookie baseball cards: 1982 *Topps* (book value $70), 1982 *Donruss* ($40), and 1982 *Fleer* ($40) cards as well as a sheet of University of Wyoming basketball trading cards distributed to fans in attendance of "Midnight Madness" at the Arena-Auditorium on the campus of the University of Wyoming in October, 1994. We consider the *Topps* card to be an imperfect substitute for the *Fleer* and *Donruss* cards, and therefore cannot expect to compare bids cleanly between the *Topps* and the other two card types. On the other hand, based on several market factors we consider the *Fleer* and *Donruss* cards to be perfect substitutes: the two cards appear to have had very similar quantities printed, their book values at the grade level we use show a one-for-one relationship over the past nineteen years, and each card was independently graded as "PSA 8 near-mint" by a well-known agency, Professional Sports Authenticators (PSA).[23] Within each auction treatment we displayed the same good to bidders, and identical copies were sold to winning bidders.

Table 5 presents the experimental treatments based on (i) number of bidders—2, 3, or 5; (ii) bidder experience—professional card dealers or non-dealers; and (iii) auction type—uniform-price or Vickrey auction. The table also introduces the mnemonics we will use to refer to the different treatments: for sportscards, D2 for two dealers per auction, D3 for three dealers per auction, ND5 for five nondealers per auction, and so on. D3Sheet and D5Sheet denote the auctions for the University of Wyoming basketball trading card sheets. In our design, we were careful to facilitate clean comparisons across bidder types and numbers of bidders. For example, we ensured clean comparisons of Vickrey to uniform-price auctions within each row of the

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but the data were noisier. In the new treatments, we chose to auction only high-value goods (book value at least $20).

[23] The 1982 *Topps* cards and the 1989 Barry Sanders *Score* card, from the earlier treatments of List and Lucking-Reiley (2000), were also graded 8 by PSA.

table—e.g., the Vickrey and uniform-price auctions in D3 both use the same *Topps* card. And, across rows of the table, ND3, D5, and ND5, as well as D3sheet and D5sheet treatments, can safely be compared to each other, because they auctioned the same, or highly similar goods. The bid levels in the D2, ND2, and D3 treatments are neither directly comparable to the other treatments nor to each other.[24] Our goal in this particular experimental design was to measure the amount of demand reduction (between auction formats) in each different setting, and to compare how the amount of demand reduction changes with the number of bidders.

Each bidder in each auction went through a four-step procedure. In Step 1, the monitor invited potential subjects to participate in an auction for the two goods (participation rates were similar across auction types). The potential subject was told that the auction would take about five minutes. He (or she) could pick up and visually examine each good, which was sealed with the appropriate grade clearly marked (in the case of the cards). Nobody bid in more than one auction. In Step 2, the monitor gave the bidder two printed materials: (i) an auction rules sheet that included a practice worksheet, and (ii) a bidding sheet. The instructions were identical across treatments, except for auction type and number of bidders.[25] The instructions explained how the auction worked, provided examples to illustrate the auction, and asked the bidder to work through an example himself to make sure he understood the auction rules. Each bidder was told that he would be randomly grouped with two or four other bidders of the same type—dealers

[24] Since we auctioned the identical *Topps* card in D2 as in D3, one might have hoped this would provide an opportunity for a clean comparison. Unfortunately, the environment appears to have changed significantly between the $n = 2$ and $n = \{3,5\}$ treatments. Specifically, whereas the two-person auctions took place in Orlando, FL in June, 1998 (in the early part of the baseball season), our new three- and five- person auctions took place after the baseball season, during November, 1998 (the middle of the football season) in Tampa, FL. Because of Ripken's and the Baltimore Orioles' subpar 1998 seasons, consumer demand for Ripken cards was much lower in November than in June, and this change is not reflected in the book values as reported by PSA (which tend to be sticky downwards). Indeed, Table 6 shows that mean bids of all types (first-unit or second-unit, Vickrey or uniform-price) were considerably lower in the November auctions (3 dealers) than in the June auctions (2 dealers) for the same card, but we believe these effects should be ascribed to changes in the environment rather than changes in the number of bidders.

[25] Instructions for both the three-bidder and five-bidder auctions in the Vickrey and uniform-price institutions may be found at <http://uaeller.eller.arizona.edu/~reiley/papers/VaryingBidders.html>.

matched with dealers; non-dealers matched with non-dealers. The auction treatment was changed at the top of each hour. In Step 3, the participant submitted two bids on a bidding sheet. In Step 4, the monitor explained the ending rules of the auction, how the winners would be determined, and how and when the exchange of money for goods would occur.

The dealer treatments were identical to the nondealer treatments, except that the monitor visited each dealer at his/her booth before the sportscard show opened. All new sportscard auctions were held at a regional sportscard show in Tampa, FL, in November 1998, and the sheet of basketball trading card treatments were held at a regional sportscard show in Tucson, AZ, in March 2001.[26] A total of 366 subjects participated in our new treatments.

4. Applicability of the Modeling Assumptions to the Sportscard Market

Our theoretical model makes several important assumptions. Specifically, we assume marginal values are decreasing, independent across bidders, and privately known. We now examine the consequences of these assumptions and argue that they are reasonable assumptions for our field setting.

First, let us consider the assumption of decreasing marginal values. Actually, the mathematics appear to be valid for the case of "flat demands," or more precisely, the case in which $v_{i1} = v_{i2}$ with probability one. In this case, however, there may be multiple equilibria.[27] In one of these equilibria, everyone bids truthfully; each bidder submits two truthful bids and there is no demand reduction. Hence, in this case the theory allows for zero demand reduction as a possible

[26] Results from the 2-person auctions are taken from List and Lucking-Reiley (2000). These data were gathered at a different sportscard show, in June 1998. The $n = 2$ subject instructions are virtually identical to those for $n = 3$ and $n = 5$, except that the uniform-price treatment used only one example for the uniform-price auction, rather than three.

[27] See, e.g., Ausubel and Cramton (2002) and Engelbrecht-Wiggans (1999).

outcome. We should note, however, that this is a knife-edge case—as soon as demands become even slightly marginally decreasing, the truth telling equilibrium disappears.[28]

Our observations of the sportscard market lead us to believe that decreasing marginal values is a reasonable assumption for most bidders' demands in the card auctions. A typical non-dealer might want one copy of the card for his collection, but a second copy might be desirable only for resale. Dealers typically buy cards from consumers at only half their retail price, so consumers might well find that they value the second card less than the first. Since dealers are involved in resale, one might well expect dealers to have flatter demands than non-dealers, but we believe that even most dealers' demands are likely to be at least somewhat marginally decreasing. Recall from the discussion above that we obtain our data from regional card shows. Dealers at such shows tend to be relatively small operators, with slim margins, low overhead (making money only at weekend card shows rather than having a card shop), and relatively small card stocks. They fear being caught on the wrong side of the market, so they attempt to remain liquid and diversify their card stock. At these particular shows, as well as most other smaller regional shows, we observed that most of the dealers had only one copy of key cards, and that dealers who did have more than one copy offered quantity discounts to increase volume.[29] We therefore strongly believe that in our environment, most of our bidders valued winning two copies of a card less than twice the value of winning just one card. Note, however, that this assumption would be invalid if data were gathered at a national card show, where many large-scale dealers would essentially have flat demands. In the case of the basketball card sheets, we are even more inclined to believe that dealers have downward sloping

[28] For the case of an infinitely-divisible good, Ausubel and Cramton (2002) provide general conditions under which demand reduction is guaranteed to exist.
[29] This characteristic of dealer demands also is prevalent in card shows in the Midwest (Wisconsin, Illinois, Iowa, and Michigan), where one of the authors was a card dealer from 1987-1993.

demand—the sheets are illiquid and therefore winning dealers typically keep the sheets for their own collection. This was verified via results from a questionnaire given after the bids were placed, in which over 97% of dealers stated that they would personally consume the sheets if they were deemed a winner.

Yet, it remains possible that some bidders did not have decreasing demands. In this case, one might expect these bidders to bid the same amount on both units (see, for example, Chakraborty 1998). In practice, around a quarter of our experimental subjects bid the same on both sportscards, and only 7 of 120 subjects bid the same amount on both sheets (see Table 7 below). Provided a significant fraction of bidders have decreasing demands, any bidder should expect her rivals' bids on average to be marginally decreasing. Therefore, our theory should be robust to the inclusion of some flat-demand bidders in the population.

Second, consider the assumption of independent, privately known (IPV) values. Without this assumption, equilibrium bids might behave quite differently than we predict. For example, in the case of common values, the winner's curse may make bids in both types of auction decrease as the number of bidders increases.[30] While values certainly might be affiliated, our hope is that via our experimental design, which considerably varies the probability that the good will be individually consumed or immediately put up for sale, we may be dealing with independent values in some of our treatments. But, since we have not solved a model with affiliated values, we are in a position to only speculate about what might happen when we relax the IPV assumption. Nevertheless, the IPV assumption makes for a tractable model, allowing us to produce comparative-static predictions about varying numbers of bidders in a uniform-price auction. As

[30] Pinske and Tan (2000) provide an example with affiliated privately known values in which equilibrium bids decrease as the number of bidders grows large enough, but this is again a single object pay-your-bid auction rather than a multi-unit uniform price auction.

such, we do not claim to prove that our assumptions hold in our setting.[31] Rather, we merely claim that they are a reasonable approximation to reality. Most importantly, we claim that it is of interest to see how well the resulting predictions stack up against what we observe in the field, by measuring observable outcomes rather than unobservable model primitives.

5. Results

To put our findings into perspective, we first review the results of List and Lucking-Reiley (2000), for two-unit, two-person sealed-bid auctions. Having auctioned both low-valued cards ($3 book value) and high-valued cards ($70 book value), they found statistically significant evidence of demand reduction for the high-valued cards. Second-unit bids were considerably lower in the uniform-price auctions than in the Vickrey auctions: the difference in means was between eleven and twelve dollars, or 31% to 54% of the mean second-bid level. Furthermore, the uniform-price auction treatment resulted in significantly more zero bids, and the bid reductions were large enough to cause significant changes in the allocation of goods. They also found an anomalous result, conflicting with the theoretical predictions of Engelbrecht-Wiggans and Kahn (1998): first-unit bids were higher in uniform-price auctions than in Vickrey auctions.

The remainder of this section is organized as follows. First, we examine first-unit bids, and show that the anomaly found by List and Lucking-Reiley (2000) disappears in auctions with more than two bidders. The next three sections look at demand reduction by comparing second-unit bids across auction formats: mean second-unit bids in section 5.2, proportions of second-unit bids equal to zero in section 5.3, and the overall distribution of second-unit bids in section 5.4. In section 5.5, we examine individual bid schedules across auction formats, comparing first-unit and second-unit bids submitted by the same individual. Section 5.6 moves from data on

[31] Yet our data do provide information concerning this assumption. Note that in an IPV setting demand reduction is predicted to decrease as n increases. While in a common value setting demand reduction should (probably) increase as n increases (submitting the marginal bid is "bad news" and leads to lower bidding in equilibrium).

individual bidder strategies to data on group outcomes: auction revenues, and allocations of goods. We summarize the experimental results in Section 5.7.

5.1. Mean first-unit bids

We begin by examining first-unit bids, displayed in the first two columns of Table 6. Our theory predicts the distribution of first-unit bids to be equivalent across auction formats, because it is always a dominant strategy to bid one's value on the first unit. The first two rows demonstrate the violation of this prediction discovered by List and Lucking-Reiley (2000) for two-bidder auctions. First-unit bids were significantly higher in the uniform-price than in the Vickrey auctions, by more than \$13 in the dealer treatment ($t = 3.22$) and \$10 in the nondealer treatment ($t = 1.76$). In three other treatments for low-valued cards (not reproduced here), List and Lucking-Reiley found every point estimate indicating higher first-unit bids in the uniform-price auctions, though these differences were not statistically significant.

By contrast, our new data show no such effect. Consistent with theory, first-unit bids are very similar across auction formats in each treatment. The mean bid is higher in the Vickrey auction for two treatments (D3, ND5), but higher in the uniform-price auction for the other four treatments (ND3, D3Sheet, D5, D5Sheet). None of these Vickrey-uniform differences is statistically significant ($t_{D3} = -0.20$; $t_{ND3} = 0.22$; $t_{D3sheet} = 0.58$; $t_{D5} = 0.34$; $t_{ND5} = 0.07$; $t_{D3sheet} = -0.22$). The first-bid anomaly identified by List and Lucking-Reiley (2000) appears to be special to the case of two bidders; with three to five bidders, the first-unit bid data support the theory.[32]

[32] List and Lucking-Reiley's (2000) anomalous result has been found to be prominent in 2-bidder laboratory experiments as well (see, e.g., Engelmann and Grimm, 2002), thus it does not appear a consequence specific to their field environment. This leads to the natural question of why the first-bid differences disappear with more than two bidders. Levin (2003) proposes an equilibrium for the 2-bidder case in which the bidders typically bid more than their value, and notes that similar bidder behavior is not an equilibrium when there are more than two bidders. He suggests that this may be an appropriate model, explaining why the over bidding that List and Lucking-Reiley observed in the two bidder case does not recur in the data herein. However, Levin's equilibrium does not fit the data better than the equilibria that we consider in the current paper.

Our theory also predicts that the distribution of first-unit bids should be constant with *n* in both the Vickrey auction and the uniform-price auction. Considering the *Donruss* and *Fleer* cards, we compare nondealers' bids in the $n = 3$ versus $n = 5$ treatments (ND3 versus ND5). In addition, a second clean comparison is dealers' bids in the sheet treatments (D3Sheet versus D5Sheet).[33] Data in Table 6 suggest that the distributions of first-unit bids in both the Vickrey and uniform-price auctions are independent of *n*. As *n* increases from 3 to 5, the mean first-unit bid moves from $20.03 to $20.77 ($8.10 to $7.18) in the Vickrey sportscard (trading sheet) auctions, and from $20.68 to $19.44 ($8.82 to 8.22) in the uniform-price sportscard (trading sheet) auctions. Each of these differences is statistically insignificant at conventional levels.

Overall, first-unit bids for our new data appear to be independent of both the auction format and of the number of bidders. These results are consistent with bidders understanding the rules of the auction institutions and bidding according to their dominant strategies.

5.2. Mean second-unit bids

Columns 3 and 4 of Table 6 generally provide support for the theoretical prediction of second-unit bids being lower on average in uniform-price than in Vickrey auctions. This occurs in five of the six new treatments; the exception is the treatment with five nondealers per sportscard auction (ND5). In the remaining five treatments, mean Vickrey second-unit bids exceed the corresponding uniform-price bids by as much as 23%. Yet, individual t-tests indicate that each of these differences is statistically insignificant at conventional levels ($t_{D3} = 1.15$; $t_{ND3} = 0.44$; $t_{D3sheet} = 0.73$; $t_{D5} = 0.53$; $t_{ND5} = -0.24$; $t_{D5sheet} = 0.05$). By contrast, the previous data on two-bidder auctions, shown in the first two rows of Table 6, show statistically significant differences ($t_{D2} = 3.10$; $t_{ND2} = 2.82$). These results indicate that demand reduction effects on

[33] As noted above, none of the other comparisons across bidders is clean. ND2 versus ND3 involved auctions of completely different cards, as did ND2 versus ND5. D3 auctioned a more expensive card than did D5, while D2 and D3 auctioned the same card but in very different environments.

mean second-unit bids become smaller and harder to detect as the number of bidders increases beyond two.[34]

A comparison of second-unit bids between the $n = 3$ and $n = 5$ treatments provides additional evidence on changes in demand reduction with increasing numbers of bidders. Recall that our theory predicts that as n increases, the distribution of second-unit bids remains constant in the Vickrey auction while growing weakly higher in the uniform-price auction. For comparing mean bids across numbers of bidders, we focus on comparisons between treatments: ND3 versus ND5 and D3Sheet versus D5Sheet. Results are consistent with our theoretical prediction: in the Vickrey auctions, as n changes from 3 to 5 in the card (sheet) auctions, the mean second-unit bid increases (decreases) from \$9.69 to \$9.77 (\$4.38 to \$4.22), a statistically insignificant difference, consistent with the theoretical prediction of no change.[35] In the uniform-price auctions, mean second-unit bids change considerably more, from \$8.63 to \$10.48 (\$3.77 to \$4.17) in the card (sheet) auctions. Though not statistically significant, these point estimates represent an increase of approximately 20% (10%), in the same direction as predicted by the theory.

5.3. Zero second-unit bids

Another measure of demand reduction is the proportion of zero second-unit bids (full demand reduction). Nash equilibrium theory predicts more zero bids on second units in the uniform-price auction than in the Vickrey auction. Columns 1 and 2 of Table 7 show that the

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[34] Another possible explanation for the diminished demand reduction is the slightly lower stakes in the $n > 2$ treatments relative to the $n = 2$ treatments. The $n = 2$ auctions reported here were for cards with book values of \$70, while five of the six $n > 2$ auctions were for goods with book values of \$40 or less. Indeed, List and Lucking-Reiley (2000) reported that \$70 cards generated statistically significant demand reduction, while \$3 cards did not. While lower stakes may be part of the explanation for our results, we note that \$40 and \$20 stakes are still relatively large, and believe our evidence indicates that changes in the number of bidders do have an important effect.

[35] By contrast, note that there *are* statistically significant differences between the bids of dealers and nondealers. In 5-bidder auctions, both dealers and non-dealers bid for the identical 1982 Cal Ripken *Fleer* card. The dealers' second-unit Vickrey mean bid is \$20.68 while the nondealers' is \$9.77, which is statistically significant (t = 3.24). The differences are similar in magnitude for second-unit bids in the uniform-price auctions (t = 2.34). Both differences are consistent with dealers having higher demands for the cards on average. Hence, while we fail to reject equality in mean bids across number of bidders (when the theory is consistent with the null hypothesis), the same t-test is powerful enough to reject equality across bidder types.

proportion of zero bids increases in each treatment when we move from Vickrey to the uniform price auction treatment. While in each of the eight treatments the proportion of zero bids increases from the Vickrey to the uniform-price auction, none of the increases is individually statistically significant at conventional levels. Yet, combining the six new treatments yields an aggregate t-statistic of 2.64, which is statistically significant at the $p < .01$ level.[36] Thus, there is some statistical evidence that demand reduction persists when $n > 2$.

Next, we examine how the zero-bid data vary for different values of *n*. Again, we test this hypothesis by comparing between treatments: ND3 versus ND5 and D3Sheet versus D5Sheet. Our theory makes two predictions: first, the proportion of zero bids in the Vickrey auction is independent of *n*, and second, the proportion of zero second unit bids in the uniform-price auction is weakly decreasing in *n*. Table 7 shows that the proportion of zero second-unit bids decreases (increases) in the Vickrey card (sheet) auctions: from 31% to 27% (13% to 17%). Neither difference is statistically significant. In the uniform-price auctions, the proportion of zero second-unit bids decreases from 42% to 40% and 33% to 23% in the card and sheet auctions; again, neither difference is statistically significant. For the uniform-price data, this is consistent with a value distribution for which $v^*(n)$ either remains constant, or changes very slowly, with *n*.

5.4. Distribution of second-unit bids

[36] There are two different ways to consider pooling the data across treatments. The first is to compute the aggregate proportion of zero bids in the Vickrey and uniform-price auctions, and to compute the standard z-statistic to test for a difference in proportions. The aggregate proportions of zero bids across the six new treatments are 19.7% for the Vickrey auction, and 29.5% for the uniform-price auction, which produces a z-statistic of 2.18. However, we choose to use a more powerful aggregate test statistic (see Bushe and Kennedy 1984 and Christie 1990) that takes into account the fact that the underlying proportions of zero bids may vary across treatments, even though they might still be equal (under the null hypothesis) between the Vickrey and uniform-price formats. For example, the value distributions might be different across different cards, and nondealers tend to submit higher proportions of zero bids than dealers. This new test relies on the fact that the t-test statistic for each individual treatment has an approximately normal distribution with mean 0 and variance 1. Furthermore, the six tests are all independent. The sum of six independent, normally distributed random variables is itself a normally distributed random variable with mean equal to the sum of the means and variance equal to the sum of the variances. Thus, the aggregate t-statistic equals the sum of the six independent t-statistics divided by the square root of 6, which in our case equals 2.64.

Our theoretical results indicate that there are two qualitatively different types of demand reduction in Nash equilibrium. First, low-value bidders may reduce their second-unit bids from small, positive amounts in the Vickrey auction to zero bids in the uniform-price auction. Second, high-value bidders may lower their second-unit bids in the Vickrey auction to some lower, but still nonzero value in the uniform-price auction. Both types of demand reduction appear to be present in the data, at least in seven of the eight treatments. Table 8 displays the raw data for second-unit bids in all eight treatments.

In section 5.3, we presented evidence of the first type of demand reduction: bids reduced all the way to zero. In each of the eight treatments, the number of zero bids is higher in the uniform-price auction than in the corresponding Vickrey auction, which can clearly be seen in Table 8. We should also note that, consistent with our theory as illustrated in Figure 1, the uniform-price bid distributions tend to have a gap between zero and the first positive bid amount, relative to the Vickrey bid distributions. The sharpest example is the ND3 treatment, which has four Vickrey bids of $1, while the lowest positive uniform-price bid is $3. Comparing the distributions, we find that the $1 Vickrey bids all match with $0 bids in the uniform-price auction. Using our theory's notation, it looks as if $c(v^*)$ equals $3, so that no bids are observed in the uniform-price auction below this amount. The other treatments show a similar pattern concerning low positive bids. In particular, ND2 has Vickrey bids of $2 and $3, while the lowest positive bid in the uniform-price auction is $5. D5 had three Vickrey bids of $1 or $2, but the lowest positive uniform-price bid in the uniform-price auction is $4. And ND5 shows six Vickrey bids between $1 and $4, while the lowest positive uniform-price bid is $5. The D2 and D3 treatments' lowest positive uniform-price bids precisely equal their lowest positive Vickrey bids ($10 and $3, respectively), which is consistent with the theoretical case of $v^* = 0$.[37]

Table 8 also shows evidence of the second type of demand reduction: high bids being reduced to somewhat lower, positive bids.[38] For the D2 treatment, 29 of the uniform-price bids are positive. Comparing these 29 bids to the corresponding 29 bids in the Vickrey bid distribution, we find that 27 of the bids are lower than their Vickrey counterparts (on the same row of the table), while none are higher. Thus, the empirical distribution of uniform-price bids is first-order stochastically dominated by the empirical distribution of Vickrey bids. The other cases are similar, though not quite as striking as in the D2 treatment. For example, of 22 positive uniform-price bids in D3, 14 are lower and only 3 are higher than the corresponding Vickrey bids. Of 21 positive uniform-price bids in ND3, 13 are lower and only 4 are higher than their counterparts in the Vickrey bid distribution. And, of 24 positive uniform-price bids in D5, 14 are lower and only 1 higher than their Vickrey counterparts. In sum, the rightmost part of the bid distribution does tend to shift from higher to lower positive bid amounts when moving to the uniform-price auction.

5.5. Individual bid schedules

Another type of evidence of demand reduction involves the slopes of the bid schedules submitted by the individual bidders. In the uniform-price auction, we expect bidders' bid schedules to be more steeply sloped than they are in the Vickrey auction. That is, the difference

[37] While this is true, it is important to point out that the number of zero bids is larger (but not significantly so) in the uniform cases, contradicting $v^* = 0$.

[38] We do not know how to conduct an appropriate formal hypothesis test of this claim. A Kolmogorov-Smirnoff test detects differences in distributions, but it would have low power in this situation, where gaps between the distributions are small but persistent (it measures only the maximum gap). A t-test might be able to demonstrate a significant difference between the means of the distributions once the distributions were truncated to eliminate zero bids in the uniform-price auction, but we are unaware of a statistical technique designed to compare two distributions both truncated at an endogenously determined location. With this nuance in mind, at the request of a reviewer we carried out a series of Mann-Whitney tests on all positive bids. This particular procedure may well be considered quite careful since all positive bids in the Vickrey treatment are included, even those that appear to correspond to zero bids in the uniform treatment. We find that in the two bidder auctions we can reject the homogeneity null for both the dealer and nondealer data at the $p < .05$ level. Yet we can never reject the null at conventional levels in the three or five bidder treatments.

between a bidder's first bid and his second bid should be greater under the uniform-price than under the Vickrey auction format. The mean differences between a bidder's first and second unit bids show this pattern in all treatments but ND5, as can be inferred from the bid statistics in Table 6. For example, in the D2 treatment from the previous paper, the uniform-price bid difference of $32.07 far exceeds the Vickrey bid difference of $7.83. The corresponding comparisons are $15.73 versus $10.48 (D3), $12.05 versus $10.34 (ND3), $5.05 versus $3.72 (D3Sheet), $12.82 versus $10.44 (D5), and $4.05 versus $3.66 (D5Sheet). The effects are in the predicted direction (except for the case of ND5), though the effects become smaller as the number of bidders increases. In the six new treatments, none of the differences is statistically significant.

The middle columns of Table 7 report data on the proportion of individuals who submitted flat bid schedules, with first bid equal to second bid. For example, in the ND3 treatment the Vickrey auction exhibits 19.4% flat bid schedules, while the uniform-price auction exhibits only 11.1%. While there are some observed differences, they tend to be quite sporadic: we observe more flat bid schedules in the Vickrey auction for treatments D2, ND2, D3, and ND3, but less flat bid schedules in the Vickrey auction for D3Sheet, ND5, and D5Sheet. The effects are statistically significant in the $n = 2$ treatments, but not in the new $n = 3$ and $n = 5$ treatments. Yet, in general the differences tend to be smaller when the number of bidders increases.

Our data on bid schedules also illustrate an important difference between dealers and nondealers. In the five-bidder sportscard treatments, the identical *Fleer* card was auctioned to both dealers and nondealers. Dealers submitted flat bid schedules in both auction formats (approximately 30% of the time) more often than nondealers (approximately 20%). Though this difference is not quite statistically significant in isolation, a similar pattern exists for the other sportscard treatments, even though the auctioned cards in those treatments were not the same between dealers and nondealers. This confirms the intuition that we had when we originally designed the experiment: dealers' demands for cards tend to be flatter than nondealers' demands. Our reasoning was that while dealers have resale opportunities for multiple identical cards, nondealers' resale opportunities are limited, and they may be more interested in just a single copy for their collections. The data in Table 6 also indicate another difference between bidder types: dealers bid larger amounts than nondealers on average. For the five-bidder treatments, where the comparison is clean, dealers bid approximately \$30 and \$20 on average for first and second units, while nondealers bid only \$20 and \$10.

5.6. Revenues and allocations

Having considered evidence on individual bidding decisions, we now turn to group outcomes: revenues and allocations of goods. Since bidders submitted their bids independently, we compute averages of these group outcomes over all possible combinations of players, using the recombinant technique discussed in Mullin and Reiley (2005). The rightmost columns of Table 6 present descriptive statistics on revenues in our new auction treatments. As in the $n = 2$ experimental treatments reported in an earlier paper, we find no consistent revenue ranking between the auction formats. Mean uniform-price revenues were higher than mean Vickrey revenues in five of the six new treatments, with D3 the exception. However, these revenue differences are not statistically significant for any of the individual treatments, either individually or as a group (see footnote 39).

The presence of demand reduction can lead, in theory, to inefficient allocations of goods. In particular, when one bidder has the two highest values for each of two units, but submits a strategically low bid on the second unit, the second unit may be allocated inefficiently to another bidder. The previous experimental treatments for $n = 2$ showed that demand reduction was large enough to cause measurable changes in allocations: significantly more auctions resulted in the two units being split between two bidders in the uniform-price auctions than in the Vickrey auctions. The rightmost columns of Table 7 compare those allocation results to the new results for the $n = 3$ and $n = 5$ treatments. In the new treatments, the proportion of split allocations differs in the predicted direction for all but the ND5 treatment. Though none of the results of the new treatments are individually significant, the results are collectively significant for the treatments where $n = 3$.[39] Thus, as the number of bidders increases from 2 to 3 to 5, it becomes increasingly difficult to detect a difference in split allocations between the Vickrey and uniform-price formats.

5.7. Summary of experimental results

The preceding sections have provided a variety of experimental results. Here, we summarize them for convenient reference:

1. The unexpected first-unit-bid differences discovered in the two-bidder case disappear when the number of bidders increases to three or more.
2. Demand reduction in mean second-unit bids also diminishes when the number of bidders increases, but not as quickly as the first-unit effect disappears.
3. The proportion of zero bids remains significantly higher in the uniform-price auction than in the Vickrey auction for $n > 2$.
4. There is evidence of both types of theoretically predicted demand reduction: small positive bids become zero, and large positive bids become smaller positive bids, when moving from the Vickrey to the uniform-price auction.

[39] The test statistics on differences between proportions of split allocations for treatments D3, ND3, and D3Sheet are z = 1.76, 1.35, and .781 respectively. Since these test statistics are themselves independent standard-normal random variables, their sum is a normal random variable with mean zero and variance 3. Therefore, to test the joint null hypothesis that the proportions of split allocations are equal across auction formats, an appropriate standard-normal test statistic is the sum of the three z-statistics divided by the square root of 3. In this case, the aggregate test statistic is z = 2.25, indicating that collectively speaking, the proportions of split allocations are significantly different for the treatments with 3-bidder groups. This same technique does not yield a significant aggregate test statistic for the 5-bidder treatments.

5. Our data are generally consistent with the comparative-static predictions of our theory. As n increases from 3 to 5, first-unit mean bids remain approximately constant. Second-unit mean bids also remain approximately constant, consistent with a theory that predicts bids to be either constant or increasing in n. We should stress that these failures to reject null hypotheses might, of course, result from type-II error. Yet, we note that for the same bid data, our measurements are precise enough to find statistically significant differences between the mean bids of dealers and nondealers for the same card.
6. Allocations continue to differ in the predicted direction between the Vickrey and uniform-price auctions, although we should stress that the effect is only collectively statistically significant for $n = 3$ and not at all significant for $n = 5$.
7. The revenues generated are approximately equal between auction formats. This equivalence holds for all n from 2 to 5.
8. The treatment with five nondealer bidders produced results that contrasted with theoretical predictions and with the results of all the other experimental treatments. In the ND5 treatment, the distribution of positive second-unit bids was higher, rather than lower, in the uniform-price relative to the Vickrey auction. However, the difference in mean bids was not statistically significant, and the proportion of zero bids did change in the expected direction.

6. Concluding Remarks

In this paper, we have presented new theory and new experimental evidence on bidding behavior in multi-unit auctions. We have provided theoretical arguments that for a fixed number of units to be auctioned, an increase in the number of bidders causes weakly diminished incentives for demand reduction in second-unit bids. Though diminished with increasing n, some demand reduction does persist in the asymptotic limit. In our field experiment, while the data do not adhere to every conjecture, in general we find evidence consonant with many of the theoretical predictions. We note that demand reduction remains present with three to five bidders per auction, but it becomes smaller and less statistically significant than in previous results with only two bidders per auction.

Demand reduction is a topic of considerable practical relevance, with multi-unit auctions increasingly being utilized in the economy. Examples include recent government auctions for spectrum rights and pollution permits, auctions in recently deregulated electricity markets, and the emergence of multi-unit auctions on the Internet for objects from laser printers to Beanie Babies. The investment-banking firm OpenIPO has even begun to conduct initial public offerings of stock shares via uniform-price sealed-bid auctions on the Internet. Previous economic research has shown that strategic demand reduction is a serious concern that may cause large inefficiencies, both in theory and in practice. These previous findings suggested using Vickrey auctions in place of uniform-price auctions, despite the increased complexity and the potential for perceived "unfairness" of the mechanism.

Yet the evidence is not quite as clear in the realistic case of more than two bidders. True, we find that demand reduction persists, but the amount of demand reduction seems to decrease dramatically. Indeed, we found it relatively difficult to find statistically significant evidence of demand reduction for more than two bidders. Furthermore, for any fixed amount of demand reduction in individuals' bids, the probability that a second-unit bid (rather than a first-unit bid) determines the price will decrease as the number of bidders increases. Thus, we might expect demand reduction to have a very limited effect when there are more than a few bidders.[40]

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[40] Indeed, as Swinkels (1996) demonstrates, the uniform-price auction is efficient in the limit as the number of bidders goes to infinity.

While interesting, we have thus far considered only the case of two units being auctioned.[41] In many real-world applications, such as auctions for Treasury debt or for electric power, the number of units is likely to be considerably greater than two. We might expect demand-reduction incentives to increase with the number of units, because a lower bid on the price-determining marginal unit may then provide the bidder with the benefit of a lower price on *multiple* inframarginal units. Indeed, in practical applications such as Treasury debt and electric power, the number of units may well be larger than the number of bidders. Further research remains to be done on the problem of demand reduction with varying number of units.

[41] This was done for reasons of theoretical tractability. In this model, the dominant strategy of truthtelling reduces the problem to a one-dimensional strategy: that of the second-unit bid amount. This allows us to provide a general characterization of equilibria with any distribution of independent private values. With three or more units up for auction, the problem becomes much more complicated, as second-unit and third-unit bid strategies depend on each other. For this reason, we have not yet found a way to characterize equilibria in auctions for more than two units.

Appendix: Calculation of uniform-price equilibria.

In this appendix we provide theoretical insights into how equilibria might vary with the number n of bidders. To do so, we turn to the work of Engelbrecht-Wiggans and Kahn (1998) on uniform-price auctions with privately known values. For the case of two units, Engelbrecht-Wiggans and Kahn provide conditions characterizing equilibria in undominated strategies. Examining these conditions suggests that equilibria might change with n in a very specific manner.

We start by recalling the key definitions and results from Engelbrecht-Wiggans and Kahn. In particular, Lemma 2.2 assures that any symmetric Nash equilibrium in undominated strategies has the first bid equal to the first value and the second bid $b(v_{i2})$ satisfying $0 \leq b(v_{i2}) \leq v_{i2}$ for all v_{i2}. Therefore we focus our attention on the second bid.

In the multi-unit setting, the usual first order condition for an equilibrium is intractable. Instead, Engelbrecht-Wiggans and Kahn discovered a more tractable expression that serves the purpose of characterizing the second bid at equilibrium; unfortunately they provide no intuitive explanation for this expression. In particular, they define

$$\Gamma(c,v;n) = \int_0^c (n-1)G_1(x)^{n-2}[G_1(x) - G_2(v) + g_1(x)(v-x)]dx$$

and

$$C(v;n) = \underset{c \in [0,v]}{\operatorname{argmax}} \Gamma(c,v;n)$$

where we have amended their original notation to show that Γ and C both also depend on n .

They also define $C(v;n)$ to be an "increasing correspondence" (for any fixed n) if

$$x < y,\ a \in C(x;n), \text{ and } b \in C(y;n)$$

together imply that

$$a \leq b.$$

Their Corollary 5.4 implies that, if $C(v;n)$ is an increasing correspondence and $c(v;n)$ is a selection from $C(v;n)$, then a Nash equilibrium in undominated strategies results from each bidder i bidding $(v_{i1}, c(v_{i2}))$. Furthermore, if the distribution of v_{i2} conditional on v_{i1} has support $[0, v_{i1}]$, then their Theorem 3.2 assures that for any Nash equilibrium in undominated strategies, the second bid $b(v_{i2};n)$ is strictly less than the second value v_{i2} for all $v_{i2} \in (0, V)$.

Now let us see what can be inferred from these definitions and results. First, note that Corollary 5.4 and Theorem 3.2 together imply that, if $C(v;n)$ is an increasing correspondence and $c(v;n)$ is a selection from $C(v;n)$, then $0 \leq c(v;n) < v$ for all $v \in (0, V)$. Next note that $c(v;n) \in C(v;n)$ and $0 < c(v;n) < v$ together imply that $c(v;n)$ satisfies the following first-order condition:

$$G_1(c(v)) - G_2(v) + g_1(c(v))\,(v - c(v)) = 0$$

Let $C'(v)$ denote the set of solutions to the first order condition; note that $C'(v)$ is independent of n. Then combining the two previous observations gives the following Lemma, which combined with the previously mentioned results of Engelbrecht-Wiggans and Kahn provides:

Lemma: If $C(v;n)$ is an increasing correspondence and $c(v;n)$ is a selection from $C(v;n)$, then for each $v \in (0, V)$, either $c(v;n) = 0$ or $c(v;n) \in C'(v)$.

Theorem: If $C'(v)$ is an increasing correspondence, $c'(v)$ is a selection from $C'(v)$, and there exists a cutoff $v^*(n)$ such that

$v \in (0,v^*(n))$ implies that $\Gamma(0,v;n) > \Gamma(c'(v),v;n)$ (1)

and

$v \in (v^*(n),V)$ implies that $\Gamma(0,v;n) < \Gamma(c'(v),v;n)$ (2)

then a Nash equilibrium in undominated strategies results, for the auction with n bidders, from each bidder i bidding $(v_{i1},b(v_{i2}))$, where $b(v_{i2}) = c'(v_{i2})$ for $v \in (v^*(n),V)$ and $b(v_{i2}) = 0$ otherwise.

This theorem suggests a procedure for computing equilibria. Specifically, first solve for $C'(v)$. Then, pick a $c'(v)$ from $C'(v)$; if the first order condition has a unique solution, then it determines $c'(v)$. Next fix n. Finally, compare $\Gamma(0,v;n)$ with $\Gamma(c'(v),v;n)$ for each $v \in (0, V)$ and determine whether there exists a cutoff $v^*(n)$ satisfying the conditions (1) and (2) of the Theorem.

Ideally, we would now provide general characterizations of how the cutoff $v^*(n)$ varies with n. The following two Corollaries of the Theorem establish conditions under which the cutoff–and, therefore, the second-unit bid–is independent of n:

Corollary 1: If $g_1(0)$ is infinite, then the second-unit bid will be positive with probability one. Proof: If $g_1(0)$ is infinite, then the integrand in the expression for $\Gamma(c,v;n)$ will be positive for small enough c, and therefore the maximizing c must be positive.

Corollary 2: (Corollary 4.4 of Engelbrecht-Wiggans and Kahn 1998) If $g_1(x)$ is an non-decreasing function of x, then a Nash equilibrium results when each bidder i bids $(b_{i1}, b_{i2}) = (v_{i1}, 0)$.

Unfortunately, we have been unable to provide general conditions under which $C'(v)$ is an increasing correspondence and there exists an appropriate cutoff $v^*(n)$, and then characterize how the cutoff $v^*(n)$ varies with n. Instead we turn to the examples discussed in Section 2

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Figure 1. Graph of the equilibrium bid function b(v) for second-unit bids versus second-unit values, where values come from the k=3 probability distribution (see Table 1). Separate graphs are displayed for n=2 and n=5 bidders.

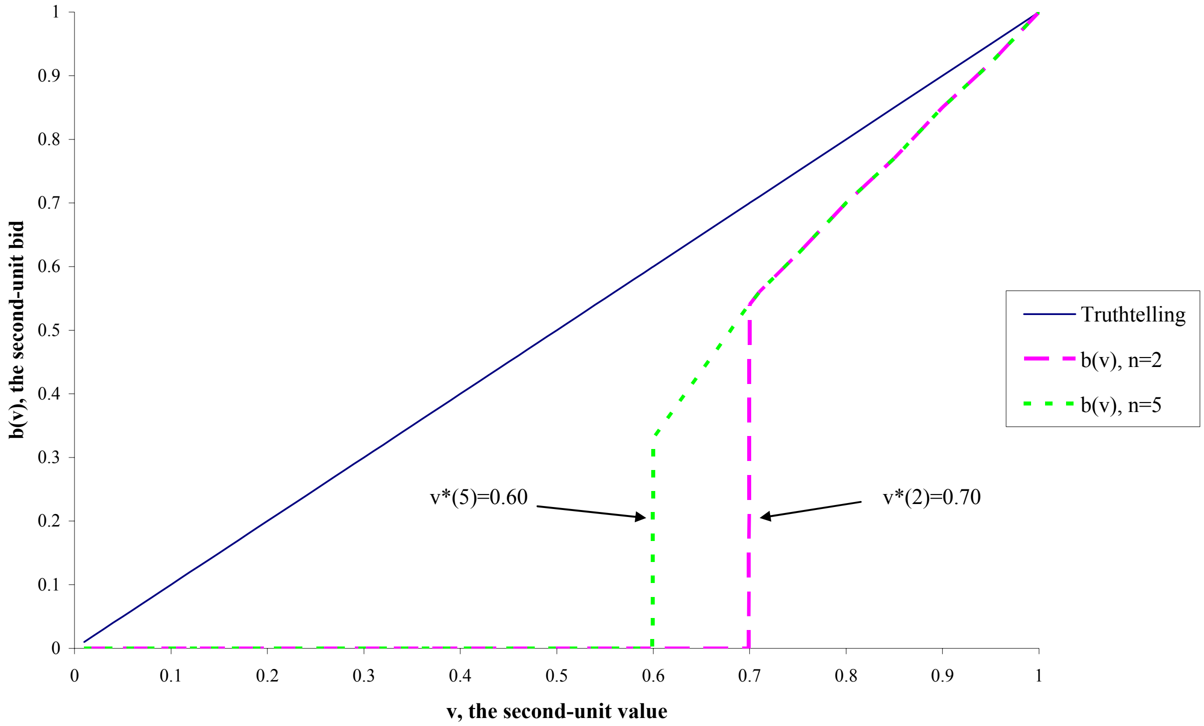

Table 1a. Cutoffs (c^*) when $H(x)$ = Beta(α_1=1, α_2) as a function of n and α_2.

	n =	2	3	4	5	6	7	8	9	10
α_2 =	≤ 1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	0.93	0.81	0.79	0.79	0.77	0.77	0.77	0.77	0.77
	3	0.67	0.60	0.60	0.58	0.58	0.58	0.58	0.58	0.58
	5	0.44	0.39	0.39	0.38	0.38	0.38	0.38	0.38	0.38
	7	0.34	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.28
	10	0.24	0.21	0.20	0.20	0.20	0.20	0.20	0.20	0.20

Table 1b. Pr(b_2 = 0) when $H(x)$ = Beta(α_1=1, α_2) as a function of n and α_2.

	n =	2	3	4	5	6	7	8	9	10
α_2 =	≤ 1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.000	0.999	0.998	0.998	0.998	0.998	0.998	0.998	0.998
	3	0.999	0.996	0.996	0.995	0.995	0.995	0.995	0.995	0.995
	5	0.998	0.994	0.994	0.992	0.992	0.992	0.992	0.992	0.992
	7	0.998	0.992	0.992	0.992	0.992	0.992	0.992	0.992	0.990
	10	0.996	0.992	0.990	0.990	0.990	0.990	0.990	0.990	0.990

Table 2a. Lower bound on cutoff (c^*) when $H(x)$ = Beta(α_1, α_2) for various α_1 and α_2.

	α_1 =	< ½	1	2	3	5	7	10
α_2 =	≤ 1	0.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	0.00	0.77	0.89	0.93	0.95	0.97	0.98
	3	0.00	0.58	0.75	0.82	0.88	0.91	0.94
	5	0.00	0.38	0.55	0.65	0.75	0.80	0.85
	7	0.00	0.28	0.44	0.53	0.64	0.71	0.77
	10	0.00	0.20	0.33	0.41	0.52	0.60	0.67

Table 2b. Lower bound on Pr(b_2 = 0) when $H(x)$ = Beta(α_1, α_2) for various α_1 and α_2.

	α_1 =	< ½	1	2	3	5	7	10
α_2 =	≤ 1	0.000	1.000	1.000	1.000	1.000	1.000	1.000
	2	0.000	0.998	0.999	0.999	1.000	0.999	1.000
	3	0.000	0.995	0.997	0.998	0.998	0.998	0.999
	5	0.000	0.992	0.995	0.997	0.998	0.998	0.998
	7	0.000	0.990	0.995	0.996	0.997	0.997	0.997
	10	0.000	0.990	0.994	0.995	0.995	0.995	0.996

Table 3. E[$b_2(v_2)/v_2$] when $H(x)$ = Beta(α_1=⅓, α_2) for various α_2 (independent of n).

α_2 =	⅓	1	2	3	5	7	10
E[$b_2(v_2)/v_2$] =	0.001	0.014	0.019	0.022	0.022	0.022	0.022

Table 4a. Lower bounds on cutoff (c*) and Pr(b_2 = 0) when $H(x)$ = Gamma(α, β=1) for various α.

α =	< 0.5	0.5	0.6	0.7	0.8	0.9	1	2	3	4	5	7	10
c^* >	0	0.736	1.14	1.43	1.74	1.99	2.25	3.95	5.73	7.35	9.32	12.48	18
Pr(b_2 = 0) >	0	0.952	0.974	0.980	0.986	0.988	0.990	0.992	0.995	0.996	0.998	0.999	0.9999

Table 4b. E[$b_2(v_2)/v_2$] when $H(x)$ = Gamma(α, β=1) for various α (independent of n).

α =	0.1	0.2	0.3	0.4	0.4	0.5
E[$b_2(v_2)/v_2$] =	0.028	0.033	0.026	0.018	0.018	0.012

Table 5. Experimental Treatments.

Treatment	Good auctioned	Book value	Bidders per auction	Bidder type	Vickrey auctions	Uniform-price auctions	Total subjects
D2	Ripken *Topps*	$70	2	Dealers	15	15	60
ND2	Sanders *Score*	$70	2	Nondealers	17	17	68
D3	Ripken *Topps*	$70	3	Dealers	9	9	54
ND3	Ripken *Donruss*	$40	3	Nondealers	12	12	72
D3Sheet	UWYO sheet	$20	3	Dealers	10	10	60
D5	Ripken *Fleer*	$40	5	Dealers	6	6	60
ND5	Ripken *Fleer*	$40	5	Nondealers	6	6	60
D5Sheet	UWYO sheet	$20	5	Dealers	6	6	60

Notes:

1. Treatments D2 and ND2 took place in May 1998, and were previously reported in List and Lucking-Reiley (2000). The other four sportscard treatments, designed specifically for the present paper, took place in November 1998. The final two UWYO sheet treatments, again designed for this study, took place in March 2001.
2. The total number of subjects includes all number of bidders who participated in either auction format. For example, in treatment ND3, there were 36 bidders in the 12 Vickrey auctions and 36 more bidders in the 12 uniform-price auctions, for a grand total of 72 subjects.

Table 6. Summary Statistics.

	Bid on Unit #1		Bid on Unit #2		Total Revenue	
Treatment	Vickrey	Uniform	Vickrey	Uniform	Vickrey	Uniform
D2	49.60 (15.19)	62.67 (15.28)	41.77 (14.46)	30.60 (13.43)	73.52 (25.42)	74.30 (21.00)
ND2	51.82 (23.44)	62.21 (25.32)	28.82 (19.98)	16.62 (15.40)	49.17 (20.88)	45.25 (27.59)
D3	39.74 (26.87)	38.17 (26.44)	29.26 (25.05)	22.44 (17.81)	64.54 (32.37)	61.93 (27.63)
ND3	20.03 (13.60)	20.68 (13.46)	9.69 (10.46)	8.63 (9.94)	27.76 (14.70)	30.82 (14.13)
D3Sheet	8.10 (4.42)	8.82 (4.87)	4.38 (2.94)	3.77 (3.48)	11.96 (3.57)	13.36 (5.07)
D5	31.12 (22.81)	31.45 (15.91)	20.68 (14.82)	18.63 (15.00)	67.90 (19.23)	72.30 (16.53)
ND5	20.77 (14.20)	19.44 (15.93)	9.77 (11.03)	10.48 (11.74)	44.33 (14.50)	46.89 (19.96)
D5Sheet	7.88 (5.36)	8.22 (6.39)	4.22 (3.86)	4.17 (4.37)	15.57 (5.28)	18.31 (6.44)

Notes:
1. Treatment names (D2, ND3, etc.) are as defined in Table 5.
2. The table presents sample means, with sample standard deviations in parentheses.
3. Bid #1 and Bid #2 are the first and second bid submitted by a bidder.
4. "Total Revenue" is the total payment received for both goods in the auction.
5. Total Revenue data represent recombinant means and standard deviations.

Table 7. Proportions of zero bids, flat bid schedules, and split allocations.

	Zero second-unit bids		Flat bid schedules		Split allocations	
Treatment	Vickrey	Uniform	Vickrey	Uniform	Vickrey	Uniform
D2	0.0%	3.3%	36.7%	6.7%	30.34%	88.28%
ND2	5.9%	20.6%	14.7%	0.0%	52.50%	86.99%
D3	11.1%	14.8%	25.9%	18.5%	30.14%	47.08%
ND3	30.6%	41.7%	19.4%	11.1%	54.73%	68.10%
D3Sheet	13.3%	33.3%	3.3%	6.6%	67.55%	75.04%
D5	6.7%	20.0%	30.0%	30.0%	65.25%	68.38%
ND5	26.7%	40.0%	20.0%	23.3%	72.30%	62.83%
D5Sheet	16.7%	23.3%	6.6%	9.9%	61.60%	64.62%

Notes:
1. Treatment names (D2, ND3, etc.) are as defined in Table 5.
2. "Zero second-unit bids" indicates the proportion of second-unit bids equal to zero.
3. We omit data on first-unit bids equal to zero, because only one bidder bid zero on the first unit in any of these treatments. (This was a bid in the uniform-price auction in the ND5 treatment.)
4. "Flat bid schedules" denotes the proportion of bidders whose bid schedules are flat (first-unit bid equals second-unit bid).
5. "Split allocations" indicates the proportion of auctions for which the two goods were split between two bidders.
6. We used the recombinant technique to find the proportion of split allocations.

Table 8. The empirical distributions of second-unit bids.

D2		ND2		D3		ND3		D3Sheet		D5		ND5		D5Sheet	
V	U	V	U	V	U	V	U	V	U	V	U	V	U	V	U
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	10	2	0	0	0	0	0	0	0	1	0	0	0	0	0
25	15	3	0	3	0	0	0	0	0	1	0	0	0	0	0
25	20	5	0	3	3	0	0	1	0	2	0	0	0	0	0
25	20	10	0	5	5	0	0	2	0	5	0	0	0	1	0
25	20	10	0	9	10	0	0	2	0	8	4	0	0	1	0
25	25	10	5	10	10	0	0	2	0	8	7	0	0	1	1
30	25	10	5	10	10	0	0	3	0	8	8	1	0	2	1
30	25	20	10	10	14	0	0	3	0	10	8	1	0	2	1
35	25	20	10	15	15	0	0	3	2	10	10	1	0	2	1
40	25	20	10	15	18	1	0	3	3	15	10	3	0	2	2
40	25	20	10	20	20	1	0	4	3	15	10	3	5	3	2
43	30	20	10	25	20	1	0	4.5	3	20	15	4	5	3	2.5
45	30	25	11	35	20	1	0	5	4	20	18	5	6	3	3
45	30	25	14	35	23	4	3	5	4	23	20	5	10	3	3
50	30	25	15	40	25	5	5	5	5	25	20	10	10	4	4
50	32	25	15	40	25	5	7	5	5	25	24	10	10	4.5	4
50	32	25	20	45	30	5	8	5	5	25	25	10	12	5	5
50	35	30	20	45	30	5	10	5	5	25	25	10	12	5	5
50	40	30	20	50	32	10	10	5	5	25	25	10	15	5	5
55	40	30	20	50	40	10	10	5	5	30	25	15	20	5	5
55	40	30	20	50	45	10	10	5	5	35	25	20	20	5	5.5
55	40	34	20	50	46	15	10	6	5	35	30	20	20	6	8
55	40	45	20	60	50	15	12	6	6	35	35	20	20	8	8
55	45	45	20	65	50	20	15	8	7	40	40	25	25	9	9
60	45	48	25	100	65	20	15	9	9	40	40	25	25	10	10
60	49	50	25			20	15	10	10	40	45	25	25	10	10
60	55	50	25			20	16	10	10	45	45	30	30	12	12
60	60	55	30			21	20	10	12	50	45	40	45	15	18
		60	30			25	20								
		60	30			25	20								
		68	50			25	20								
		70	75			25	20								
						30	20								
						30	45								

Notes:
1. Treatment names (D2, ND3, etc.) are as defined in Table 2.
2. V denotes a Vickrey auction, while U denotes a uniform-price auction.
3. Each column presents the raw data on second-unit bids from a given auction treatment, sorted in ascending order.